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DCL-003-1164004

Seat No. ________

M. Sc. (Sem. IV) Examination

July – 2022

Mathematics : CMT-4004

(Graph Theory)

Faculty Code : 003
Subject Code : 1164004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.

(2) There are total five questions.

(3) Each question carries equal marks. (14)

1 Answer any **seven** questions : **7×2=14**

(1) Define terms : Null graph and simple graph. Also draw a simple null graph G, with $|V(G)|=5$.

(2) Define terms : The complete graph K_n, The complete bipartite graph $K_{m,n}$.

(3) Define terms : Walk, Trail, Path and Cycle in a graph (Circuit in a graph G).

(4) Write down atleast three properties of adjacency matrix of a simple graph.

(5) Define term : Coloring of a graph and chromatic number of a graph.

(6) Write down atleast three properties for chromatic number of some graph.

(7) Define distance between two vertices in a connected graph.

(8) Define term : Eccentricity of a vertex in a connected graph and center of connected graph.

(9) Define term : Diameter of a connected graph.

(10) Define term : Weighted graph and minimal spanning tree.

2 Answer following **two** questions : **2×7=14**

(1) Let $G=(V,E)$ be a graph. Prove that, G is a disconnected graph if and only if there are two disjoint subsets V_1 and V_2 of V such that, (i) $V=V_1 \cup V_2$ and (ii) there is no edge uv in G, whose one end vertex lies in V_1 and another end vertex lies in V_2.

(2) Let G be a connected graph. Prove that, G is a tree if and only if by adding an edge between its any two non-adjacent vertices creates exactly one cycle.

(3) Let G be a connected graph with n vertices and $S \subseteq E(G)$. Prove that, S is a cut-set for G if and only if rank of $G-S$ is $n-2$ and rank of $G-S_1$ is $n-1$, for every S_1 proper subset of S.

3 Answer any **one** question : **1×14=14**

(1) Define term : Ring sum of two cut sets of a connected graph G. For a connected graph G, prove that, the ring sum of two cut-sets of G is either a cut-set of G or it is an edge disjoint union of two cut-sets of G.

(2) For a simple connected planner graph G, derive Euler's formula $f=e-n+2$ and also prove that, (i) $e \geq \frac{3f}{2}$ (ii) $e \leq 3n-6$. Using these, prove that, K_5 and $K_{3,3}$ both are non-planner graphs, where e = number of edges, n = number of vertices and f = number of faces in the planner graph G.

4 Answer any **two** questions : **2×7=14**

(a) Let $S \subseteq E(G)$ be a subset in a graph G and $S \cap E(T) \neq \phi$, for all the spanning trees T of G and no proper subset of S has above property. Prove that, S is a cut-set for the graph G.

(b) Let G be a connected graph and S be a cut-set for G. Let F be any cycle in G. Prove that, $|E(F) \cap S|$ is even.

(c) Let G be a connected graph and S is a cut-set for G. Let T be a spanning tree for G. Prove that, $S \cap E(T) \neq \phi$.

5 Answer any **two** questions : **2×7=14**

(i) Let u and v be distinct vertices of a tree T. Prove that, there is a unique path P between u and v in T.

(ii) Let G be an acyclic graph with n vertices and k components. Prove that, G has $n-k$ edges.

(iii) Define acyclic graph. Let G be an acyclic graph and G satisfies $|E(G)|=|V(G)|-1$. Prove that, G is a connected graph or it is a tree.

(iv) Let T be a tree and it has atleast two vertices. Let $P = u_0 - u_1 - u_2 - ...... - u_n$ be the longest path in T. Prove that, u_0 and u_n both are pendent vertices in T.